**Quantity, Extension, and Infinity in the 13th and 14th Centuries**

**Abstracts**

**Expanding quantity: Robert Grosseteste on the bodily form**

Nicola Polloni (KU Leuven)

Robert Grosseteste (d. 1253) shows a marked tendency to explain physical and metaphysical theories by recurring to the geometrical scheme of light radiation. Moving along geometrical lines, light displays its causal agency when interacting with the bodies and their ontological root, i.e., matter. Bodies are the primary subject of a universe that is moved like a clockwork: a “machina mundi” that can be moved only insofar as its cogs are extended, quantified substances. Yet before quantity and extension, before bodiliness and physicality, the cosmological roots of the universe were an extensionless substrate and an agent that tends towards infinite extension: prime matter and light. How can quantifiable bodies come out of an extensionless, featureless prime matter? In my paper, I will analyse Grosseteste’s theory of the bodily form as bestower of extension (i.e., potentially quantified bodiliness). First, I will discuss the kind of hylomorphism endorsed by Grosseteste, particularly whether he adheres to universal hylomorphism or not. Second, I will examine how he envisioned the functionality of the bodily form and what implications his theory has in terms of formal pluralism. Finally, I will draw some general conclusions on Grosseteste’s hylomorphism.

**The unity of matter, extension and potentiality in late thirteenth century natural philosophy**

José Filipe Silva (University of Helsinki)

In the second half of the thirteenth century, thinkers delved into the question of the unity of matter – literally, whether matter is one (numerically or otherwise) – that is to say, whether there is a matter common to celestial and terrestrial bodies, and common to spiritual and corporeal beings, and what is the meaning of ‘common’ here. Although this debate has been noted by other scholars, my paper aims at providing new insights into how the question about the unity of matter relates to the issues of extension and the nature of matter as the substrate for change. As the notion of potentiality is central to this issue, I also aim at understanding what is the relation (if any) between unity of matter and potentiality. I will do this by focusing on Robert Kilwardby, Roger Bacon, and Geoffrey of Aspall, but is not limited to them.

**Numbers and diagrams: Shao Yong (1012–1077) and his legacy in 11th–13th century China**

The Aristotelian category of quantity, translated as *jihe* 几何 (“how much/how many”), was introduced to China during the 17th century first through the translations of the Italian Jesuit, Matteo Ricci (1552-1610), then through the Chinese translation of the Coimbra commentary on the *Dialectics* (*In Universam Dialecticam*, 1606) by the Portuguese Jesuit Francisco Furtado (1589-1653) and the Chinese Christian Li Zhizao 李之藻 (1571-1630). The latter source, titled in Chinese *Mingli tan* 名理探 (*Investigation of the Pattern of Names*, 1636-1639), presents a very detailed and elaborate treatment of both continuous and discrete quantity. Intriguingly, its influence on intellectual discussions in China, where the philosophic orientation was moral and practical and less concerned with the realm of logic, was minimal. The general preoccupation of Chinese philosophy with questions of morality and human conduct influenced the perception of indigenous philosophical ideas as well, resulting in marginalization of certain native philosophical schools. This was the case of the so-called school of images and numbers (*xiangshu xuepai* 象数学派) that originated with the scholarship of Shao Yong 邵雍 (1012-1077). Shao developed a system of classification of the universe based on numbers (*shu* 数), which embraced not only the realm of material objects, but also language and time. My paper will first briefly present Shao Yong’s ideas on numbers and his classification of the universe and then concentrate on the interpretation and development of these ideas by his followers during the 11th-13th centuries.

**The substance and function (*ti-yong*) formula and Aristotle’s hylomorphic model**

Gad C. Isay (Tel Hai College)

This study introduces a Confucian perspective – specifically ideas of Zhu Xi (1130-1200) – for the purpose of triggering the formulation of new understandings in this Occident-centered discussion. The Confucian conception of the material world is based on an understanding of *qi* (already not spirit and not yet matter) in relations with *li* (pattern). This *qi* is self-creating and self-generating and is non-quantifiable. While atoms increase or decrease in quantity, Zhu Xi proposes the *qi* that condenses and expands. The latter’s worldview moreover, is categorically non-polar and boundless, and events are not conceived as caused by one specific preceding event, but as woven in a network of interdependent nodes, a colossal pattern in which things react upon each other by a kind of resonance rather than mechanical impulsion. After presenting an outline of the relevant aspects of Zhu Xi’s worldview my discussion dwells on his account for the relatedness between the one and the many, thereafter concentrating on a comparative analysis of Aristotle’s hylomorphic (matter and form) model and Zhu Xi’s understanding of the substance and function, *ti-yong*, formula.

**Fakhr al-Din al-Razi on the category of quantity**

Ahmed Igbariah (Tel Aviv University)

Fakhr al-Din al-Razi’s (d. 1210) theory of categories is undertaken from an ontological point of view, while his logical works have no reference to it. This theory is surveyed in his philosophical work *al-Mabahith*, as an extension of the discussion of possible existents. He divides these existents into substance and accident, which are the categories, and discusses them ontologically since they are deduced from the concept of ‘existence’. In *al-Mabahith*, the discussion of the categories is considerably longer than in any philosopher prior to al-Razi. This lengthening is the result of al-Razi’s reworking of the theory of the categories, especially the four first ones: substance, quantity, quality, and relation. An examination of the category of quantity has increased dramatically compared with the brief discussion in Aristotle *Categories*. In my paper I will argue that, for al-Razi, the category of quantity is the main category after the category of substance and it plays an important role in understanding the natural world.

**Continuity, Limits, and Quantity in Al-Farabi's Paraphrase of Aristotle's *Categories* and its Citations in Samuel Ibn Tibbon's *Explanation of Foreign Terms***

Yehuda Halper (Bar Ilan University)

Averroes repeatedly accused Al-Farabi of denying the possibility of human conjunction (*ittiṣāl*) with the incorporeal Active Intellect, even though Al-Farabi actually affirms the possibility of conjunction in his political writings. In fact, the possibility of conjunction does not fit simply into Al-Farabi's logic or natural philosophy, largely because he does not distinguish between conjunction and continuity (both *ittiṣāl* in Arabic). Moreover, for Al-Farabi continuity is really a part of quality. Al-Farabi defines quality as "everything of which the totality can be measured by a part of it." Clearly qualities must have parts, and thus cannot have the kind of intellectual unity required for intellectual conjunction with the Active Intellect. Further, while Aristotle defined the continuous as having a shared boundary (ὅρος) at which its parts join, Al-Farabi requires a limit (*nihāya*) in addition to a boundary (*ḥadd*). This definition, which may be indebted to Euclid's 13th definition, connects continuity to the divisibility of lines, surfaces, and bodies and discusses infinite divisibility of parts with shared limits, approximately 1000 years before Cauchy. While this definition of continuity was later largely ignored, Samuel Ibn Tibbon preserves it in his 1213 Hebrew work, *Explanation of Foreign Terms*, where he is also reluctant to endorse intellectual continuity.

**From *Horror vacui* to *amor infini*: The context and function of ornamentation considering the notions – void, infinity, and the place of God**

Danielle Omesi Moisa (Tel Aviv University)

At the turn of the 13th century, the conception of God's infinite power gave way to the notion of infinity as *the* most important characteristic of God's being. This shift in the definition of divine reality, from infinite power perpetually yielding an infinite number of creatures, to infinity as the very essence of God's being – “God is an infinite sphere whose center is everywhere and his circumference nowhere” (Alan of Lille, †1202) – “infinitely removed [God] from the bondage of matter as a completely separate substance” (Richard Fishacre, †1248).

The proposed talk will explore the changing relations between architecture and ornamentation, when the most characteristic feature of "Romanesque" architecture – namely, the abhorrence of void, i.e., *horror vacui* – gave way to ornamental expansion – *amor infini* (Gombrich, 1984) – as the most characteristic expression of the High Middle Ages, imbuing sacred superstructures with a conceptual antinomy that inspired a visual experience of divine infinity.

**Measures in the Lucca Codex: Following instructions and making mistakes**

Yael Barash (Tel Aviv University)

The illustrations in Lucca Codex (Bibliotheca Statale, codex 1942) are exceedingly famous within the medieval research community. They are a beautiful example of cosmological maps, with an extensive use of gold and color. The codex is the only survived medieval illustrated copy of *Liber divinum operum*, Hildegard of Bingen’s last book. It was made in 1220s-30s, half a century after Hildegard’s death. Thus, the illustrations are an early reflection of Hildegard’s text made by actors of the 13th century. By observing the expensive illustrations, it becomes clear that makers invested much effort in creating the codex. *Liber divinum operum* contains a detailed description of the current cosmos, the world of the afterlife and at the End of Time. In the talk, I will try to show how the makers of the codex tried to follow these instructions carefully. However, despite their great efforts, some mistakes were still made.

**Actual infinities of different kinds in Richard Kilvington’s philosophical and theological texts**

Robert Podkoński (University of Łódź)

Recent studies on Richard Kilvington’s philosophical and theological texts reveal that he was one of the most ingenious and influential thinkers of his times, even though his followers frequently adopted and adapted his ideas without mentioning his name. Some of his novel ideas were, hovewer, too much ahead of his times, and thus were either overlooked or harshly criticized by traditionally-minded philosophers. This was surely the fate of Kilvington’s concept of infinity. In his question *Utrum omnis creatura sit certis limitibus circumscripta* (from his commentary on the „Sentences”) Kilvington introduces distinction between absolute infinity (*simpliciter*), and relative infinities (*secundum quid*). According to him there can exist relative infinities with regard to quality (*qualitative*), quantity (*quantitative*), or number (*discretive*). A closer look at Kilvington’s analyses reveals that he considers all such infinities as actual ones – following in this respect Ockham’s concept of infinity. Some of the reasonings Kilvington included in the above-mentioned question he first discussed in the question *Utrum continuum sit divisibile in infinitum* (from his commentary on *De generatione et corruptione*). In my presentation I focus on perhaps the most sophisticated of Kilvington’s accounts on the *infinitum discretive*, that are antedating Hilbert’s and Cantor’s ideas.

**Quantity and infinity in Jacques Legrand's *Compendium utriusque philosophie*: A late nominalist approach to Aristotle**

Daniel A. Di Liscia (LMU Munich)

At the beginning of the fifteenth century, nominalism is not only alive but, as Jacques Legrand’s philosophical *Compendium* testifies, in its best shape. In this didactical work, Legrand aims to offer a presentation of the Aristotelian natural philosophy that should not only be accessible to students but also includes the main contemporary theoretical trends. Examining the key physical notions, Ockham’s razor finds application throughout Legrand's work, but only “as far as possible”. In my talk, I will revise Legrand's nominalist lines of thought on quantity and infinity, two substantial cores of Aristotelian philosophical physics which are mutually intertwined. I will focus on the arguments involving (dis-)continuity and God’s creational power. As we will see, a mere summary of old opinions was for Legrand’s purpose neither sufficient nor truly effective. Instead, he had to go a step forward and work out a special approach to be able to face the more modern objections brought up by a group of logicians, natural philosophers and theologians. In doing so, Legrand did not manage to re-invent nominalism but, at least, to update it in a still useful way.

**Richard Kilvington’s concept of measuring the qualitative changes by the latitude of forms**

Elżbietą Jung (University of Łódź)

In his *Queastiones super Physicam*, Richard Kilvington (ca. 1302-1361) discusses in detail the problems of all kinds of changes, i.e. local motion, transformations of the elements and qualitative changes of mixed bodies. As a nominalist, he recognizes that only substances and qualities are subjects to change; other eight categories are used to describe and explain these changes. Therefore, mathematics, along with logic, is a legitimate explanatory method. Some of the procedures he uses are quasi-mathematical, such as determining the first and last moments of change, or the maximum or minimum values of the factors involved in the change; others are strictly mathematical, such as the calculus of proportions and the “calculus” of equinumerous infinite sets. The second group also includes the “calculus” of latitudes. Kilvington reasons that of latitude we can talk only when a qualitative change takes place, since a latitude itself is the same as the *qualitas* being changed. In order to explain natural phenomena, he introduces a kind of “calculus of the latitudes of forms,” which in the following centuries has been called *scientia media*.

**From the measure of qualities to the quantification of metaphysical essences. On some 14th-century applications of limit decision problems to the perfection of species**

Sylvain Roudaut (Stockholm University)

The reflections on the perfection of species are one of the most original outcomes of the tendency to quantify different objects proper to the 14th century. While several studies have underlined the role of mathematics in the development of these reflections, many aspects of them have never been properly studied. The objective of this paper is twofold. First, to shed light on the origins of these reflections by showing how the concepts used to quantify species stem from the quantification techniques formerly applied to qualities. Second, to establish how this conceptual transfer led to apply limit decision problems typical of Aristotelian physics to metaphysical objects such as specific essences. Concepts regarding limits, continuity and divisibility originally employed to quantify qualities stimulated new reflections on the structure of essences and the different types of infinities involved in the hierarchy of species. The solutions provided by different thinkers like John of Mirecourt, Peter of Ceffons or John of Ripa to these limit decision problems show how these intensely debated questions gave rise to quite divergent metaphysical doctrines.

**John Buridan and the piecemeal view of substantial change**

Roberto Zambiasi (University of Torino)

It is a well-known fact that a number of early 14th-century thinkers adopted the belief that substantial forms (save for the intellective soul) have quantitative parts. Nevertheless, what has not been remarked is that, in some cases, this belief had important consequences on these thinkers' conception of substantial change. One case in point is John Buridan. Indeed, Buridan believed that the substantial change of the elements (and, ultimately, of all homogeneous substances) is a continuous process happening over an extended interval of time through the quantitative parts of their substantial forms, taken to be extended and co-located with the parts of their matter. This progressive view of substantial change is what I call the 'piecemeal view of substantial change'. In this talk, I will look in detail at the main features of Buridan's piecemeal view of substantial change, by focusing especially on Buridan's commentaries on the *Physics* and on the *De generatione et corruptione*. Moreover, I will also consider the wider implications that the piecemeal view of substantial change has for Buridan's thought, with a special focus on his theory of persistence.

**Nicholas d'Autrécourt on minima and indivisibles**

Aurélien Robert (CNRS, Université de Paris)

Nicholas of Autrécourt is probably one of the most famous medieval atomists, especially because he is the only one to admit the existence of a vacuum between atoms and who reduced the different kinds of changes to local motions of atomic particles. In this presentation, I would like to show that the similarities with Epicurean and Lucretian atomism are even stronger, as soon as one looks at the distinction Nicholas of Autrécourt makes between atoms, minima and indivisibles. Indeed, he responds to the arguments of Aristotle's Physics by distinguishing two levels of indivisibility, which could be called natural and mathematical, or physical and conceptual. I will try to show that, according to him, atoms are much like *minima naturalia*whereas indivisibles are point-like entities. Atoms are therefore corpuscules made of indivisibles. If atoms must be finite, it is possible that indivisibles exist in an infinite number in a continuum. Without this distinction, the atomistic physics defended in the first chapter of Autrecourt's *Tractatus*dedicated to the eternity of thingswould contradicts what he says in the second dedicated to indivisibles. Moreover, the arguments defending that the continuum is made of indivisibles would fail without this crucial distinction. This will be the occasion to discuss several interpretations of Nicholas of Autrécourt's atomism which, in my opinion, have not properly grasped this important distinction.

**Crescas' conception of infinite magnitudes**

Avishay Gallili (Tel Hai College)

Hasdai Crescas (1340-1410) was a Spanish rabbi from Saragossa and one of the leading figures in Jewish thought and culture. Crescas was a great Jewish Halacha scholar, leader, and, most of all, man of faith - his faith led him to become one of the most radical philosophers in Jewish history and the thought of the Middle Ages. His philosophy stemmed from his theology, and both are inseparable. His philosophy doctrine was established as a result of contradicting the Aristotelian and Maimonidean physics and metaphysics. In doing so, Crescas developed a new physical and metaphysical conception that served as the origin of the revolution in "new physics" established in the 14th century at the universities of Oxford and Paris. In his physical and metaphysical paradigm, he attributes four fundamental characteristics to space and time that will change how we conceptualize them forever: First, space and time are defined as continuous quantity. Second, space and time are both defined as separate from physical objects. Third, space and time are both defined as infinite. Fourth, the space and time of an object are conceived as intervals.

**Does the *aevum* have parts? Some 13th-century reflections on the simplicity of eviternity**

Dominic Dold (University of Notre Dame)

One of the more curious theoretical concepts of Scholastic metaphysics is the *aevum* (or eviternity): a temporal order occupying a middle ground between eternity and time proper. The proper measure of the being of angels and other spiritual substances, it has a beginning (like time), but no end (like eternity). Medieval conceptions of eviternity still pose exegetical problems and have even sparked some scholarly controversies.

In this talk, I revisit a controversy about temporal parts of spiritual substances. Richard Cross (1999) puts forward an interpretation of Bonaventure's conception of the *aevum*that identifies a form of medieval four-dimensionalism. *Pace* Cross, Damiano Costa (2020) argues that Bonaventure conceptualises spiritual substances as simple and lacking temporal parts.

I embed Bonaventure into the medieval debate, comparing his conception to those found in the *Summa Halensis*, and the writings of Robert Kilwardby and Albert the Great. On that basis, I propose an argument for Bonaventure's commitment to the claim that spiritual substances have temporal parts.

**William of Ockham on the ontological status of quantity**

Cecilia Trifogli (Oxford University)

Ockham’s parsimonious ontology of the material world only contains (individual) substances and (individual) qualities. Both these kinds of things can be quanta or extended but their being extended does not require that we posit quantity-things distinct from them. Consider a material substance, for example, wood. Ockham’s crucial claim is that wood is a quantum by its nature alone and not in virtue of quantity as a thing distinct from wood and inhering in it. Being a quantum is having parts outside parts so that distinct parts occupy different positions; but -Ockham contends- wood has parts outside parts, i.e., extended parts, by its own nature and not in virtue of a quantity-thing thought of as an accident inhering in it. A similar account holds for those qualities that are extended, like heat and whiteness. Ockham’s reductionist position about the ontological status of quantity is very original. In my talk I will present the main arguments that Ockham puts forward in its support in the third Question of his *Tractatus de* *quantitate*, which asks *Utrum corpus quod est quantitas sit res absoluta, distincta realiter a substantia*.

**Quantity, place, and perception in 13th-century Eucharistic debates**

Julia Reed (Bar Ilan University)

The use and interpretation of Aristotelian categories of substance and accidents offered Christian theologians precise but complicated conceptual tools for philosophically defending the sacramental operation of the Eucharist, namely, the change of the bread and wine into the body of Christ. In this paper, I consider some of the ways in which the major Eucharistic debates of the thirteenth century—principally in Thomas Aquinas, Giles of Rome, Duns Scotus, and William of Ockham—understood and transformed the accident of quantity to explain the Eucharist. These debates engaged many of the major philosophical and theological questions in the late medieval period, including the metaphysics of individuation, the number and relation of souls in living bodies, the reality of universals, and the roles and relations of the external and internal senses in cognition. I show how the controversial metaphysical status of quantity in these debates both furthered specific philosophical arguments and aimed to support Eucharistic theologies that appealed to the sacramental operation that was ultimately responsible for the metaphysical change from bread and wine to Christ’s real body.

**Gregory of Rimini on infinite extended quantities**

Clelia Crialesi (KU Leuven)

This paper examines the geometric arguments against indivisibilism put forth by Gregory of Rimini (1300-1358), a prominent figure in 14th-century scholasticism, in his *Questions on Peter Lombard's Sentences* (II, dist. 2, q. 2).  As a Master of Theology in Paris, Gregory played a crucial role in introducing ideas developed in Oxford – especially those of William of Ockham – to the French University.

After illustrating Gregory of Rimini’s approach to Euclidean geometry and reading his mathematical arguments against the light of Duns Scotus’, the paper delves into Gregory’s conception of geometric items through the lenses of Ockham’s nominalism. I argue that Gregory’s mathematical arguments are revealing of a specific account of the infinite divisibility of magnitudes, for he rejects the point-based structure of continua and opens up a new understanding of their composition: not points, but rather an actual infinity of extended parts is what magnitudes are composed of.

**An impossible synthesis: Geometrical atomism vs. the continuity of matter, Roger Bacon’s solution**

Yael Kedar (Tel Hai College)

As most of his contemporaries, Roger Bacon (c.1220–1292) held to the continuity and infinite divisibility of space. However, while Aristotle refuted the geometrical atomism of Plato’s *Timeus*, which identified the elements with geometrical solids, Bacon tried to maintain both Aristotelian continuous physics and Plato’s geometrical atomism. In this paper, I analyze Bacon’s arguments concerning the kinds of geometrical forms the four elements take, and how can they fit one into another so that no vacuum will ensue. Bacon finds interesting and original solutions to the question why, although the figures of the particles of matter are rectilinear, they are formed into spheres, and links the shapes of the elements with their motion in space. In accordance with his endorsement of geometrical atomism, Bacon introduces a new, non-Aristotelian definition of a place as a point in space surrounded by four or eight right angles. The point-place is filled ‘superficially’ by four angles and ‘corporeally’ by eight. I suggest considering Bacon’s definition of place as the foundation of his attempt at a geometrical analysis of space when it comes to his optics and diagrams of motions of bodies.

**How big is a red square? Two ways of accounting for the perception of quantities in the thirteenth century**

Elena Baltuta (Tel Hai College)

Imagine I draw a 2 by 2 centimetres red square. How do I perceive its area? Are my senses directly affected by the species of the area or not? Is there a species of the area? What sense perceives the area? It my talk I compare the answers of Aquinas and Bacon. The area is indirectly perceived through its redness, Aquinas thought. For Bacon perceiving the area is a more complex process that begins with my sight perceiving multiple visual qualities of the area, and ends up with my cogitative power forming a quasi-syllogism that infers the area of the red square. The thesis I try to substantiate in my talk is that, from a systematic point of view, Aquinas and Bacon had different understandings of central metaphysical concepts such as form, matter, potentiality, and actuality.

**Unquantifiable color power and discontinuous color order in Avicenna and Naṣīr al-Dīn al-Ṭūsī**

Ruizhi Ma (Humboldt University, Berlin)

Aristotle explains what color is and why there are many colors in his De sensu. In philosophical and scientific discourse in the Antiquity and Middle Ages, these two questions are often dealt with as a whole, one as the explanans, the other explanandum. In this paper, I elaborate on Avicenna’s conception of color and color order which on the one hand, inherits Aristotle’s idea that colors can be mixed out of the primitive ones, i.e. blackness and whiteness, but also goes beyond Aristotle with a different approach to color and light which gives rise to a two-dimensional color order. In Section 1, I argue that Aristotle’s color order is not a one-dimensional continuum. In Section 2, I interpret Avicenna's innovative conception of three non-crossing passages between blackness and whiteness. The number later has been expanded into five by Avicenna’s most famous follower, Naṣīr al-Dīn al-Ṭūsī in the 13th century, as I show in Section 3. In the Conclusion, I reflect on the birth of a quantifiable and three-dimensional color space and its intellectual connections to Avicenna and al-Ṭūsī.